Exam. Code : 211004
Subject Code : 4998

M.Sc. (Mathematics) 4th Semester
STATISTICS–II
Paper—MATH-587

Time Allowed—2 Hours] [Maximum Marks—100

Note :— There are **EIGHT** questions of equal marks. Candidates are required to attempt any **FOUR** questions.

1. (a) Let $X_1, ..., X_n$ be a random sample from normal distribution $N(\mu, \sigma^2)$. Prove that $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$, $S^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n}$ are independent random variables.

(b) Define F-distribution. Find its mean and variance.

2. (a) Let $X_1, ..., X_n$ be a random sample from a continuous population with pdf :

$$F_X(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the pdf of r^{th} order statistic $X_{(r)}$, hence, find the mean and variance of $X_{(r)}$, for $1 \le r \le n$.

(ii) Find the distribution of range $R = X_{(n)} - X_{(1)}$.

(b) If X has a chi-square distribution with n degree of freedom, find moment generating function of X.

3. (a) Let $X_1, ..., X_n$ be a random sample of size n from $N(\mu, \sigma^2)$ distribution, where μ and σ^2 are unknown. Find sufficient statistics of μ and σ^2.

(b) Let $X_1, ..., X_n$ be a random sample of size n from $U(-\theta, \theta)$ distribution. Find MLE, MME of θ.

(c) If the estimator T_n based on a random sample of size n is such that $E(T_n) \to \theta$ and $V(T_n) \to 0$, prove that T_n is consistent estimator of θ. Hence prove that sample mean is always consistent estimator for population mean.

4. (a) Let $X_1, ..., X_n$ be a random sample from $U(0, \theta)$, where θ is unknown. Obtain the UMP test of size α for testing $H_0 : \theta \leq 1$ versus $H_1 : \theta > 1$. Find the value of critical constant such that the UMP test is of size 0.5.

(b) State and prove Neyman-Pearson's lemma for most powerful test.

5. (a) Let $X_1, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$, where μ and σ^2 are unknown. Find the likelihood ratio test for testing $H_0 : \mu = \mu_0$ versus $H_a : \mu \neq \mu_0$.

(b) Describe the large sample test procedure for testing the significance of difference of proportions of two independent populations.

6. (a) Describe applications of a t distribution for testing the significance of (i) single mean and (ii) difference between two means by clearly stating the assumption and hypotheses involved in these.

(b) Briefly describe test of goodness of fit.

7. For the general linear model of full rank $Y = X\beta + \epsilon$, $E(\epsilon) = 0$ and $E(\epsilon\epsilon') = \sigma^2 I$, find least square estimator $\hat{\beta}$ of β. Hence, find $E(\hat{\beta}), \text{cov}(\hat{\beta})$. Also prove that $\ell'\hat{\beta}$ is best linear unbiased estimator of $\ell'\beta$.

8. (a) Give complete analysis of two way classified data for fixed effect model.

(b) Define estimability of a linear parametric function in a Gauss Markov model. State and prove a necessary and sufficient condition of estimability.